Numerical evaluation of inverse modelling methods for 1D and 3D water infiltration experiments in homogeneous soils

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Abstract
Modelling and understanding water fluxes in the vadose zone are important with regards to water management and require appropriate characterization methods of soil hydraulic properties. The present work studies three common methods for characterization of soil hydraulic properties based on the inverse modelling of water infiltration experiments at zero pressure head at surface (Beerkan method): the CI method for Cumulative Information method and two BEST methods for Beerkan Estimation of soil pedotransfer functions These methods estimate the soil sorptivity and saturated hydraulic conductivity by fitting infiltration data using infiltration models. The CI method directly fits the experimental cumulative infiltration to the usual short time expansion of the complete analytical model proposed by Haverkamp *et al.* (1994). The BEST methods are based on a specific algorithm that splits the experimental curves into two parts, the first part being fitted to the short time expansion and the second part to the long time expansion. To test the methods, several subsets of infiltration data were generated using the complete analytical model for several radii of the disc infiltrometer source and for times ranging from zero to several truncation times. The methods were then applied and the ratio between their estimations and the target values were evaluated to quantify their related accuracy. The results clearly demonstrated that the CI method must be used only to short time infiltration data. Yet, this method is usually used without any truncation of the experimental data, whereas the truncation should certainly be required. The BEST methods proved efficient and robust, provided the steady state was reached at the end of the infiltration experiment and both short and long time data solutions were used. The gain in accuracy of the BEST methods was all the more important when the disc radius was small.

Key Words
Soil characterization, unsaturated properties, BEST method, CI method, analytical modelling, infiltration.

Introduction
Modelling and understanding water fluxes in the vadose zone are important with regards to water management. They require accurate methods for the characterization of soil unsaturated properties. Water infiltration experiments and, in particular, Beerkan experiments (with zero water pressure at the soil surface) (Braud *et al.* 2005) have become a widely used practice for obtaining soil hydraulic properties. Several methods based on inverse modelling of Beerkan water infiltration data were developed to provide the sorptivity and the saturated hydraulic conductivity. This article aims at validating several common methods (i.e., the CI method for Cumulative Infiltration method (Vandervaere *et al.* 2000) and the BEST method for Beerkan Estimation of Soil pedoTransfer function (Lassabatere *et al.* 2006; Yilmaz *et al.* 2009) by using analytically generated data and comparing the estimated and target sorptivities and saturated conductivities. Estimator accuracy was studied as a function of the data subset (i.e., very short times, short times, short times plus steady state) for several geometric configurations (large and small disc radius).

Methods
Water infiltration models
Hydraulic characterization methods are usually developed using the following analytical models pioneered by Haverkamp *et al.* (1994) applied to infiltration from a disc free-water source into a homogeneous soil:

$$I_{3D}^{O(2)}(t,S,K_s) = S\sqrt{t} + \left(\frac{2-\beta}{3}\,\Delta K + K_0 + \frac{\gamma S^2}{r_d\,\Delta\theta}\right)t \tag{1a}$$

$$I_{3D}^{+\infty}(t,S,K_s)=\left(K_s+\frac{\gamma S^2}{r_d\,\Delta\theta}\right)t+\frac{1}{2\,(1-\beta)}\ln\left(\frac{1}{\beta}\right)\frac{S^2}{\Delta K} \quad (1b)$$

$$q_{3D}^{+\infty}(S,K_s)=K_s+\frac{\gamma S^2}{r_d\,\Delta\theta} \quad (1c)$$

where S stands for sorptivity, K_s and K_0 for the saturated and initial hydraulic conductivities, ΔK and $\Delta\theta$ for the differences in conductivities and water contents between final and initial states, α and β are usually taken as 0.6 and 0.75, respectively, and r_d is the disc radius. These equations correspond to the short time (equation (1a)) and long time (equation (1b-c)) expansions of the implicit quasi-exact model proposed by Haverkamp *et al.* (1994):

$$\frac{2\Delta K^2}{S^2}t=\frac{1}{1-\beta}\left[\frac{2\Delta K}{S^2}\left(I_{3D}(t)-K_0t-\frac{\gamma S^2}{r\,\Delta\theta}t\right)-\ln\left(\frac{\exp\left(2\beta\frac{\Delta K}{S^2}\left(I_{3D}(t)-K_0t-\frac{\gamma S^2}{r\,\Delta\theta}t\right)\right)+\beta-1}{\beta}\right)\right] \quad (2)$$

Equations (1a) and (1b-c) were proved accurate provided their use was restricted to short and long time validity intervals, respectively (Lassabatere *et al.* 2009).

The CI method

The CI method consists in deriving directly the sorptivity and the saturated hydraulic conductivity from the fit of the equation (1a) with no *a priori* restriction to the experimental data. Several graphical methods were also proposed to provide additional information and validation of the use of the CI method (Vandervaere *et al.* 2000). This method is quite common and served as a basis of many characterization studies.

The BEST method

The BEST method refers to the Beerkan Estimation of Soil pedoTransfer parameters methods originally developed by Lassabatere *et al.* (2006). These authors proposed to fit the first part of the cumulative infiltration to the short time equation (1a) and the last part to the long time expansion (1c). In particular, they use the long time infiltration rate $q_{+\infty}^{\exp}$ to define the following constraint between the estimator for sorptivity ($\hat{S}$) and the estimator for the saturated hydraulic conductivity ($\hat{K}_s$):

$$\hat{K}_s=q_{+\infty}^{\exp}-\frac{\gamma\,\hat{S}^2}{r\,\Delta\theta} \quad (3)$$

Such a constraint allows the inversion of experimental data with regards to only the sorptivity $\hat{S}$; which increases the robustness of the inverse procedure (Lassabatere *et al.* 2006). Moreover, a specific algorithm allows the selection of the data that is fitted to the short time expansion (equation (1a)). The estimators $\hat{K}_s(n)$ and $\hat{S}(n)$ are estimated successively for the first n data points from five till the total number of the data points of the whole experimental dataset. Then, the maximum time $t_{\exp}(n)$ of the data set is compared to a maximum time $t_{\max}(n)$ that stands for the limit of the validity interval of the equation (1a):

$$t_{\exp}(n)\le t_{\max}(n)=\frac{1}{4(1-\beta)^2}\frac{\hat{S}^2}{\hat{K}s} \quad (4)$$

The chosen data correspond to the maximum number of points ensuring the relation (4). On that basis, Yilmaz *et al.* (2009) adapted such a method for highly sorptive soils, through considering the intercept of the long time expansion (equation (1b)) ($b_{+\infty}^{\exp}$) as a better constraint and neglecting the value for K_0, leading to:

$$\hat{K}_s=\frac{1}{2\,(1-\beta)}\ln\left(\frac{1}{\beta}\right)\frac{\hat{S}^2}{b_{+\infty}^{\exp}} \quad (5)$$

These methods differ from the CI method mainly by respecting the validity of the short time expansion *a priori* since they carry out the inverse modelling of only the short time data. The two methods developed by Lassabatere *et al.* (2006) and Yilmaz *et al.* (2009) are referred to as “BS” for the BEST Slope and “BI” for the BEST Intercept, respectively.

1 – 6 August 2010, Brisbane, Australia. Published on DVD.

Methodology of method validation
The reference data were calculated using the equation (2) on the basis of several values for the target sorptivity (S_{ref}) and for the saturated hydraulic conductivity (Ks_{ref}) and assuming K_0 is negligible (dry initial conditions). The reference data were calculated from zero to sufficient time to include very short, short and long times. The maximum time was sufficient to reach the steady state conditions, i.e., to reach a constant infiltration rate (a constant value for the derivative of the generated cumulative infiltration). Then, the whole infiltration was truncated at several times to provide different data subsets. Moreover, calculations were performed for several radii from small to large radii. For the largest, quasi infinite radius, cumulative infiltration corresponds to 1D water infiltration (equation (2) with terms containing γ being zero). The smallest radius was taken as one fifth of the scale parameter for water pressure, which is on the order of a small disc source for most soils.

Different subsets were then analysed using the CI method and the BEST methods, leading to estimated values of $\hat{Ks}$ and $\hat{S}$. Their accuracies were evaluated using the ratios between the estimated and target values:

$$R_S = \frac{\hat{S}}{S_0} \tag{6a}$$

$$R_K = \frac{\hat{Ks}}{Ks_0} \tag{6b}$$

Ratios were plotted versus the scaled maximum time of the data set (Lassabatere *et al.* 2009):

$$t^* = \frac{2\Delta K^2}{S^2} t \tag{7}$$

Results

The results show that ratio values R_S and R_K do not depend on the values of the reference sorptivity (S_{ref}) and saturated hydraulic conductivity (Ks_{ref}), provided that the data subsets were truncated at the same scaled time (equation (7)). Presented results can then be considered as the rules for any values of S_{ref} and Ks_{ref}.

For the 1D infiltration data, the results clearly show that the CI method must be restricted to the analysis of short time data. The ratios R_S and R_K greatly diverge from unity when maximum times of data subsets (t^*) increase (Figure 1). In that case the CI method leads to the under-estimation of sorptivity and over-estimation of the saturated hydraulic conductivity. Such inadequacy results from not respecting the validity intervals of the short time expansions. Fitting long time data using the short time expansion leads to miss-estimations.

Concerning the BEST methods, both methods lead to very bad estimations when small data subsets are considered. The reference sorptivity is strongly underestimated ($R_S << 1$) and the hydraulic conductivity is strongly overestimated ($R_K >> 1$). This proves that the data modelled with the BEST methods must integrate quite long time data. When this is the case, the BEST methods are much better than the usual CI method and provide very accurate estimations of sorptivity and saturated hydraulic conductivity. Such accuracy results from the specific procedure that ensures the use of the right part of the cumulative infiltration to fit the short time expansion. In addition, it may be concluded that estimations of the sorptivity are usually better than of the saturated hydraulic conductivity. In all cases, the ratios are between unity and 1.05.

Calculations performed for the 3D case are presented for the optimal use of the methods: small time data ($t^* < 0.1$) used for the CI method and both short and long time data used for the BEST methods. For the CI method, the decrease in the radius, i.e. the increase in h_g/r_d, triggers no change for the estimation of the sorptivity but worsens the estimation for the saturated hydraulic conductivity (Figure 2). For the BEST methods, the decrease in the radius decreases the ratios of estimated and target sorptivities and hydraulic conductivities. The increase in the accuracy of the BEST methods compared to the CI method is all the more important when the disc radius is small. For instance, for a disc radius r_d equal to the fifth of the scale parameter for water pressure h_g, the BI method leads to R_S and R_K ratios of 1.005 and 1, respectively.

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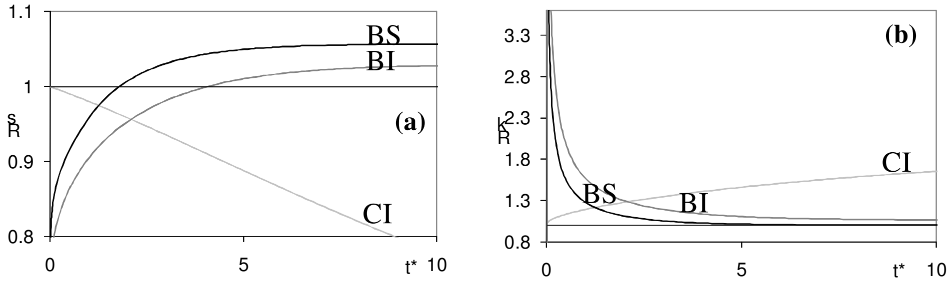

Figure 1. 1D infiltration data inverse modelling: ratios of estimated and target sorptivities –R_S- (a) and saturated hydraulic conductivities –R_K- (b) versus the scaled time (t^*).

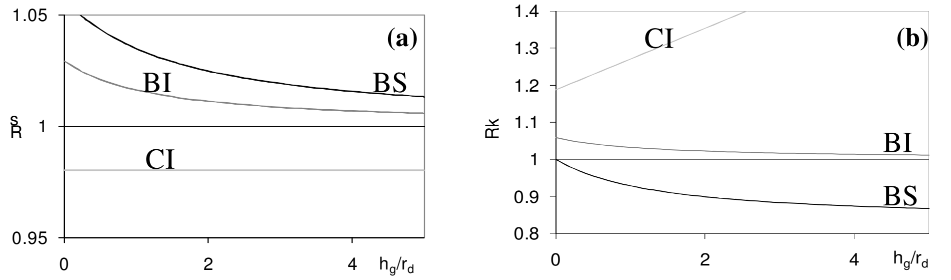

Figure 2. Ratios –R_S- (a) and –R_K- (b) versus the ratio of the scale parameter for water pressure (h_g) and the radius (r_d).

Conclusion

The presented work evaluates the accuracy of three methods involving the inverse modelling of water infiltration experiments. The CI method involves the direct fitting of the usual short time expansion of the complete infiltration model. The BEST methods fit the first part of the experimental infiltration data using the short time expansion and the second part of the infiltration data using the long time expansion. The methods include a specific procedure that splits the data into two parts. This procedure was developed to ensure the validity of the short time expansion for inverse modelling. In this study, the three methods were tested with regards to their adequacy to inversely model analytically generated reference data. The results proved that the CI method leads to an inaccurate estimation unless only the very beginning of the infiltration dataset is considered. This has a great disadvantage: only a small part of the cumulative infiltration may be used, requiring a great measurement precision for very small times. It must be noted that many methods based on the CI method (direct fitting) do not usually have any specific constraints to respect the validity of the short time expansion. On the contrary, the BEST methods provide quite accurate estimations, in particular for the sorptivity, provided that both short and long time data are used in the inverse procedure. Moreover they provide the complete set of unsaturated hydraulic parameters from the previous estimations of sorptivity and saturated hydraulic conductivity (Yilmaz *et al.* 2009). These methods appear to represent a suitable tool for the characterization of soil unsaturated hydraulic properties.

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